GENERAL INTODUCTION INTO ATMOSPHERE AND ATMOSPHERIC MOTIONS

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MUSCATEN
2nd international young scientist summer school (YSSS-2011)
OSENU, Odessa, Ukraine,
3-9 July 2011

Odessa State
Environmental University
1932

Conservation laws
for atmospheric models

- conservation of mass,
- conservation of motion,
- conservation of heat,
- conservation of water, and
- conservation of other gaseous and aerosol materials

1. Conservation of Mass

In the Earth's atmosphere, mass is assumed to have neither sink nor sources. Stated another way, this law requires that the mass into and out of an infinitesimal box must be equal to the change of mass in the box:

$$div\,(\rho\vec{V}) = {}^{\partial\rho}/_{\partial t}$$

or

$$-\ (\nabla \cdot \rho\vec{V}) = {}^{\partial\rho}/_{\partial t} \qquad (1)$$

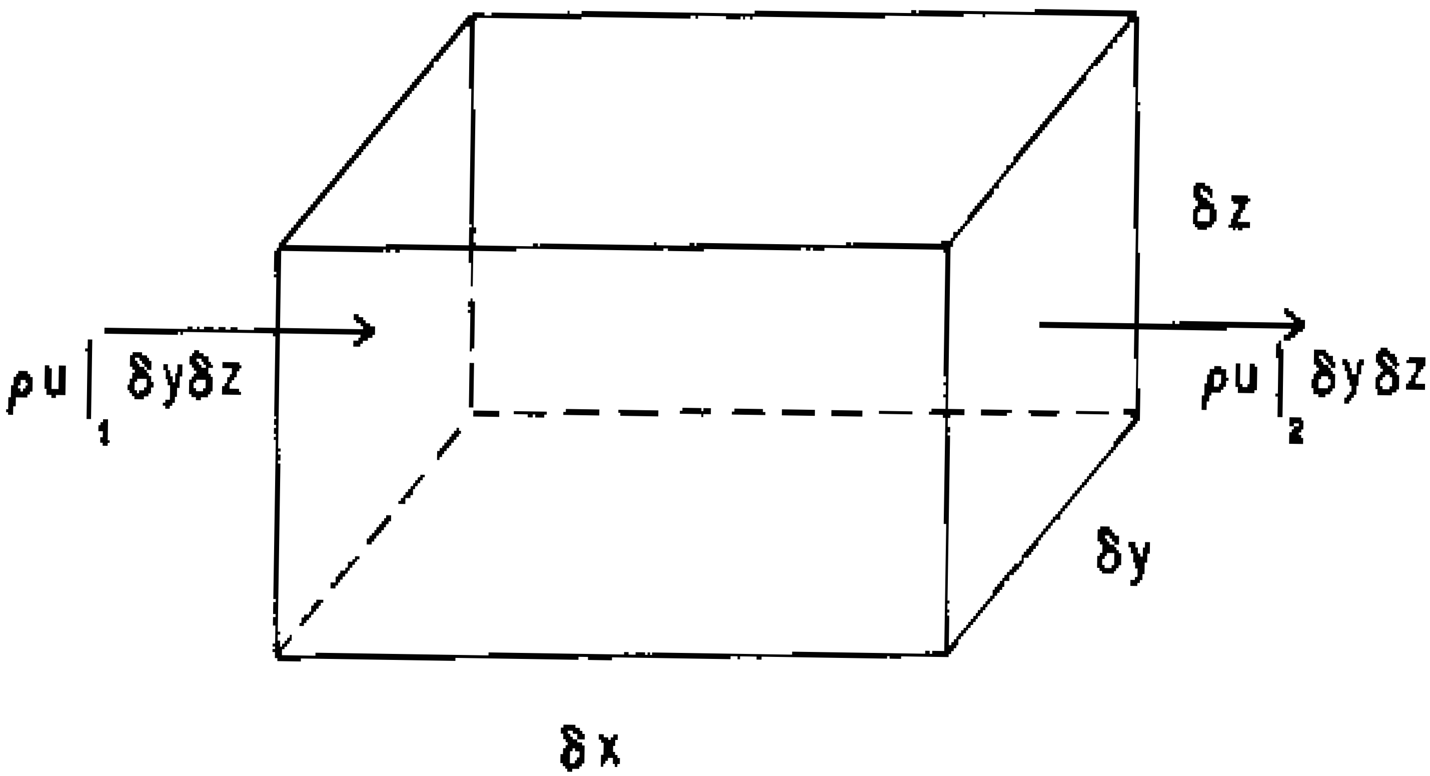

Fig. 1. *A schematic of an infinitesimal box used to derive the conservation of mass relation*

2. Conservation of Motion

The conservation of motion is expressed by Newton's second law, which states that a force exerted on an object causes an acceleration, as given by

$$\vec{F} = M\vec{a} \quad \text{or} \quad \vec{F}/_M \equiv \vec{f} = \vec{a}\,, \qquad (2)$$

where $\vec{F}$ and $\vec{a}$ are the force and acceleration vectors, respectively, and M is the mass of the object.

Equations of motion

- for laminar flows (Navier-Stokes Equations):

$$\partial\vec{V}/_{\partial t} = -\vec{V} \cdot \nabla\vec{V} - ({}^{1}/_{\rho}\, grad\, p) - g\vec{k} - 2(\vec{\Omega} \times \vec{V})$$ (3)

- for turbulent flows (Reynolds Equations):

$$\partial\vec{V}/_{\partial t} = -\vec{V} \cdot \nabla\vec{V} - ({}^{1}/_{\rho}\, grad\, p) - g\vec{k} - 2(\vec{\Omega} \times \vec{V}) + \vec{F}_{turb}$$

(4)

3. Conservation of Heat

The first law of thermodynamics for the atmosphere states that differential changes in heat content (*dQ*) are equal to the sum of differential work performed by an object (*dW*), and differential increases in internal energy (*dI*):

$$dQ = dW + dI \tag{5}$$

In a convenient form for use by meteorologists:

$$\frac{\partial \theta}{\partial t} = -\vec{V} \cdot \nabla \theta + S_{\theta} \tag{6}$$

- the definition of potential temperature: $\theta = T_V({}^{1000}/_{p})^{R_d/c_p}$

- the definition of virtual temperature: $T_V = T\,(1 + 0.61q_3)$

The source-sink term S_θ include the sum of the following processes:

$$S_\theta = \begin{bmatrix} +\text{ freezing} \\ -\text{ melting} \end{bmatrix} + \begin{bmatrix} +\text{ condensation} \\ -\text{ evaporation} \end{bmatrix} + \begin{bmatrix} +\text{ deposition (vapor to solid)} \\ -\text{ sublimation (solid to vapor)} \end{bmatrix}$$

$$+ \begin{bmatrix} +\text{ exothermical chemical reaction} \\ -\text{ endothermical chemical reaction} \end{bmatrix} + \begin{bmatrix} +\text{ net radiative flux convergence} \\ -\text{ net radiative flux divergence} \end{bmatrix}$$

$$+ \Big[\pm\text{ net turbulent flux divergence} \Big] + \begin{bmatrix} \text{dissipation of kinetic energy} \\ \text{by molecular motions} \end{bmatrix} \qquad (7)$$

4. Conservation of Water

$$\partial q_n / \partial t = -\vec{V} \cdot \nabla q_n + S_{q_n}, \quad n = 1, 2, 3 \qquad (8)$$

where q_1, q_2, and q_3 are defined as the ratio of mass of the solid, liquid, and vapor forms of water, respectively, to the mass of air in the same volume

$$S_{q_1} = \begin{bmatrix} +\text{ freezing} \\ -\text{ melting} \end{bmatrix} + \begin{bmatrix} +\text{ deposition (vapor to solid)} \\ -\text{ sublimation (solid to vapor)} \end{bmatrix} + \begin{bmatrix} +\text{ fallout from above} \\ -\text{ fallout to bellow} \end{bmatrix}$$

$$S_{q_2} = \begin{bmatrix} +\text{ melting} \\ -\text{ freezing} \end{bmatrix} + \begin{bmatrix} +\text{ condensation} \\ -\text{ evaporation} \end{bmatrix} + \begin{bmatrix} +\text{ fallout from above} \\ -\text{ fallout to bellow} \end{bmatrix}$$

$$S_{q_3} = \begin{bmatrix} +\text{ evaporation} \\ -\text{ condensation} \end{bmatrix} + \begin{bmatrix} +\text{ sublimation (solid to vapor)} \\ -\text{deposition (vapor to solid)} \end{bmatrix} \qquad (9)$$

5. Conservation of other gaseous and aerosol materials

$$\partial \varphi_m / \partial t = -\vec{V} \cdot \nabla \varphi_m + S_{\varphi_m} , \quad m = 1, 2, 3, \ldots, M \qquad (10)$$

The source-sink term $S_{\varphi m}$ includes the changes of state as well as chemical transformations, precipitation and sedimentation of important occasional constituents in the atmosphere as carbon dioxide (CO_2), methane (CH_4), sulfur dioxide (SO_2), sulfates, nitrates, ozone and herbicide and so on.

Simultaneous set of Equation of Atmospheric Dynamics in tensor notation

$$\partial\rho/_{\partial t} = -\,{}^{\partial\rho u_j}/_{\partial x_j} \quad (11)$$

$$\partial u_i/_{\partial t} = -u_j\,{}^{\partial u_i}/_{\partial x_j} - {}^{1}/_{\rho}\,{}^{\partial p}/_{\partial x_i} - g\delta_{i3} - 2\epsilon_{i,j,k}\,\Omega_j\,u_k \quad (12)$$

$$\partial\theta/_{\partial t} = -u_j\,{}^{\partial\theta}/_{\partial x_j} + S_\theta \quad (13)$$

$$\partial q_n/_{\partial t} = -u_j\,{}^{\partial q_n}/_{\partial x_j} + S_{qn} \qquad n = 1, 2, 3. \quad (14)$$

$$\partial\varphi_m/_{\partial t} = -u_j\,{}^{\partial\varphi_m}/_{\partial x_j} + S_{\varphi m} \qquad m = 1, 2, 3, \ldots, M \quad (15)$$

$$p = \rho R_d T_V \quad (16)$$